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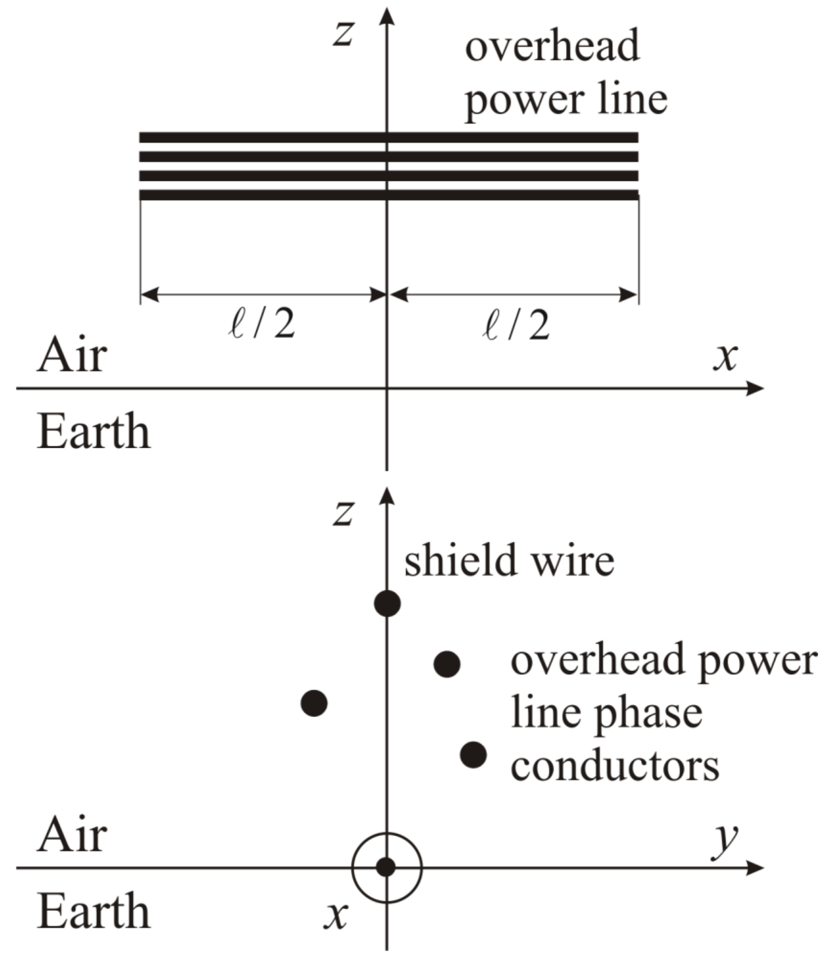
2D Computation and Measurement of Electric and Magnetic Fields of Overhead Electric Power Lines

Introduction

- the problem of computing the power frequency electromagnetic field can be considered as quasistatic → electric and magnetic field computed separately
- computation of electric and magnetic field accomplished using a 2D approximation of a power line
- short power line is considered, conductor charge density is approximated by a constant
- results will be compared to measurements underneath a 400 kV overhead power line

Mathematical model of overhead electric power line

- 2D numerical algorithm → short conductors → conductor sag neglected
- conductors positioned along *x*-axis
- conductors treated as line sources parallel to earth surface
- thin-wire approximation
- homogeneous earth
- field distribution will be computed along *y*-*z* plane in the middle of the section



Scalar electric and vector magnetic potentials

- solutions of Helmholtz differential equations reduce to solutions of **Poisson differential equations** (quasistatic approximation)
- scalar electric potential for *n* conductors:

$$\overline{\varphi} = \frac{1}{4 \cdot \pi \cdot \varepsilon_0} \cdot \sum_{i=1}^{n} \left[\int_{\Gamma_{Ai}} \frac{\overline{\lambda}_i}{R_{Ai}} \cdot d\ell_{Ai} + \overline{F} \cdot \int_{\Gamma_{Bi}} \frac{\overline{\lambda}_i}{R_{Bi}} \cdot d\ell_{Bi} \right]$$

- vector magnetic potential for *n* conductors:

$$\underline{\vec{A}} = \vec{i} \cdot \frac{\mu_0}{4 \cdot \pi} \cdot \sum_{i=1}^{n} \overline{I}_i \cdot \int_{\Gamma_{Ai}} \frac{1}{R_{Ai}} \cdot d\ell_{Ai}$$

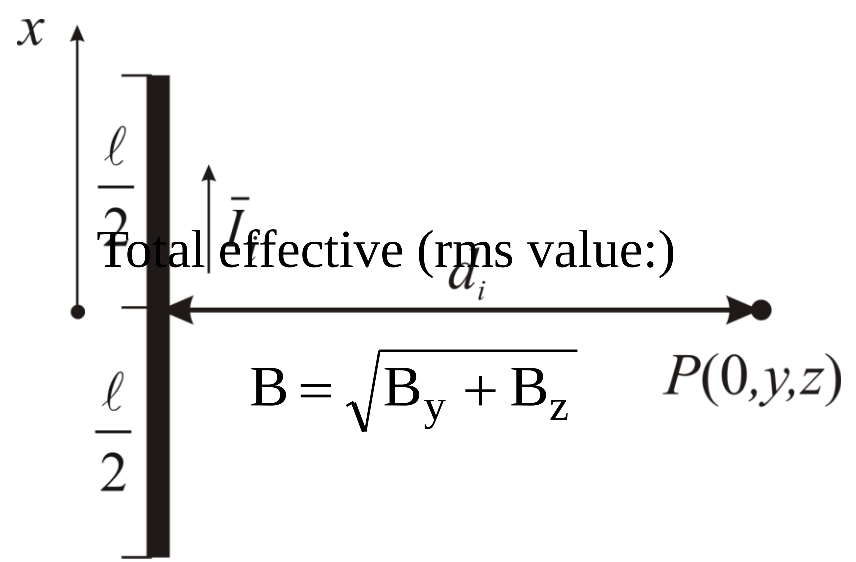
2D computation of power line magnetic field

- numerical algorithm is based on **Biot-Savart** law
- conductor current $\bar{I}_i$ flows along the axis and generates the magnetic field
- components of magnetic flux density in *y*-*z* plane for ***x* = 0**:

$$\bar{B}_x = 0$$

$$\bar{B}_y = \mu_0 \cdot \sum_{i=1}^{n} \frac{(z_i - z)\cdot \ell}{4 \cdot \pi \cdot d_i^2 \cdot \sqrt{d_i^2 + \frac{\ell^2}{4}}} \cdot \bar{I}_i$$

$$\bar{B}_z = \mu_0 \cdot \sum_{i=1}^{n} \frac{(y - y_i)\cdot \ell}{4 \cdot \pi \cdot d_i^2 \cdot \sqrt{d_i^2 + \frac{\ell^2}{4}}} \cdot \bar{I}_i$$



Total effective (rms value:)

$$B = \sqrt{B_y + B_z}$$

2D computation of power line electric field

- conductor charge density $\overline{\lambda}_i$ approximated by a **constant** $\rightarrow$ $\overline{\lambda}_i = \dfrac{Q_i}{\ell}$
- electric field intensity is computed from

$$\underline{\vec{E}} = -\nabla\overline{\varphi} - j \cdot \omega \cdot \underline{\vec{A}} = \left\{ \overline{E}_x, \overline{E}_y, \overline{E}_z \right\}$$

- the components of the electric field intensity are computed from:

$$\overline{E}_x = \overline{E}_x\big|_{x=0} = -j \cdot \omega \cdot \overline{A}_x = -j \cdot \omega \cdot \overline{A}\Big|_{x=0}$$

$$\overline{E}_y = \overline{E}_y\big|_{x=0} = -\frac{\partial\overline{\varphi}(x,y,z)}{\partial y}\Big|_{x=0} = -\frac{\partial\overline{\varphi}(0,y,z)}{\partial y}$$

$$\overline{E}_z = \overline{E}_z\big|_{x=0} = -\frac{\partial\overline{\varphi}(x,y,z)}{\partial z}\Big|_{x=0} = -\frac{\partial\overline{\varphi}(0,y,z)}{\partial z}$$

rms value:

$$\mathrm{E} = \sqrt{\mathrm{E_x} + \mathrm{E_y} + \mathrm{E_z}}$$

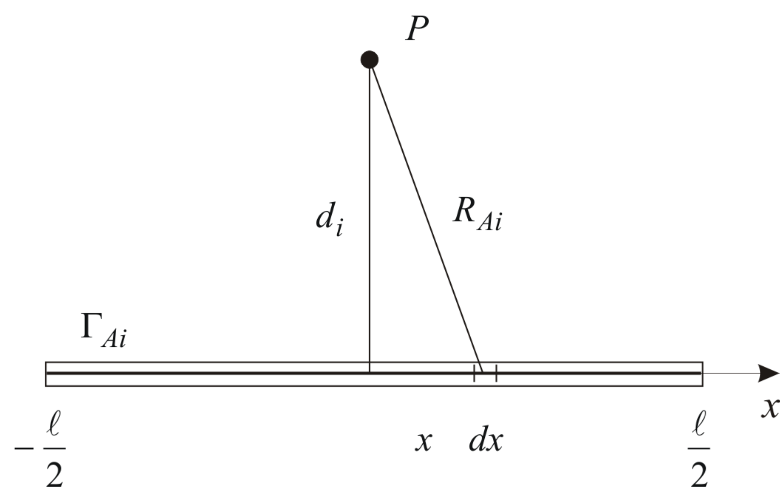
2D computation of power line electric field

➢ vector magnetic potential for $x = 0$ needed in:

$$\overline{E}_x = -j \cdot \omega \cdot \overline{A}\Big|_{x=0}$$

is computed from:

$$\overline{A}\Big|_{x=0} = \frac{\mu_0}{2 \cdot \pi} \cdot \sum_{i=1}^{n} \overline{I}_i \cdot \ell n \frac{\sqrt{d_i^2 + \frac{\ell^2}{4}} + \frac{\ell}{2}}{d_i}$$



$$d_i = \sqrt{(y - y_i)^2 + (z - z_i)^2}$$

$$\int_{\Gamma_{Ai}} \frac{1}{R_{Ai}} \cdot d\ell_{Ai}$$

2D computation of power line electric field

- scalar electric potential for $x = 0$ needed in equations

$$\overline{E}_y = -\frac{\partial \varphi(0, y, z)}{\partial y} \qquad \overline{E}_z = -\frac{\partial \varphi(0, y, z)}{\partial z}$$

can be obtained from:

$$\overline{\varphi}(0, y, z) = \frac{1}{2 \cdot \pi \cdot \varepsilon_0 \cdot \ell} \cdot \sum_{i=1}^{n} \left[\sinh^{-1}\left(\frac{\ell}{2 \cdot d_i} \right) + \overline{F} \cdot \sinh^{-1}\left(\frac{\ell}{2 \cdot D_i} \right) \right] \cdot \overline{Q}_i$$

- phasors of the i^{th} **conductor charges** are unknown

2D computation of power line electric field

- phasors of the i^{th} **conductor charges** are computed using the point collocation method:

$$\sum_{i=1}^{n}\left[\overline{Z}(d_{ji})+\overline{F}\cdot\overline{Z}(D_{ji})\right]\cdot\overline{Q}_i=\overline{\Phi}_j \quad ; \quad j=1,2,\ldots,n$$

$$\overline{Z}(v)=\frac{1}{2\cdot\pi\cdot\varepsilon_0\cdot\ell}\cdot\ell n\frac{\sqrt{\left(\frac{\ell}{2}\right)^2+v^2}+\frac{\ell}{2}}{v}$$

- v is replaced by:

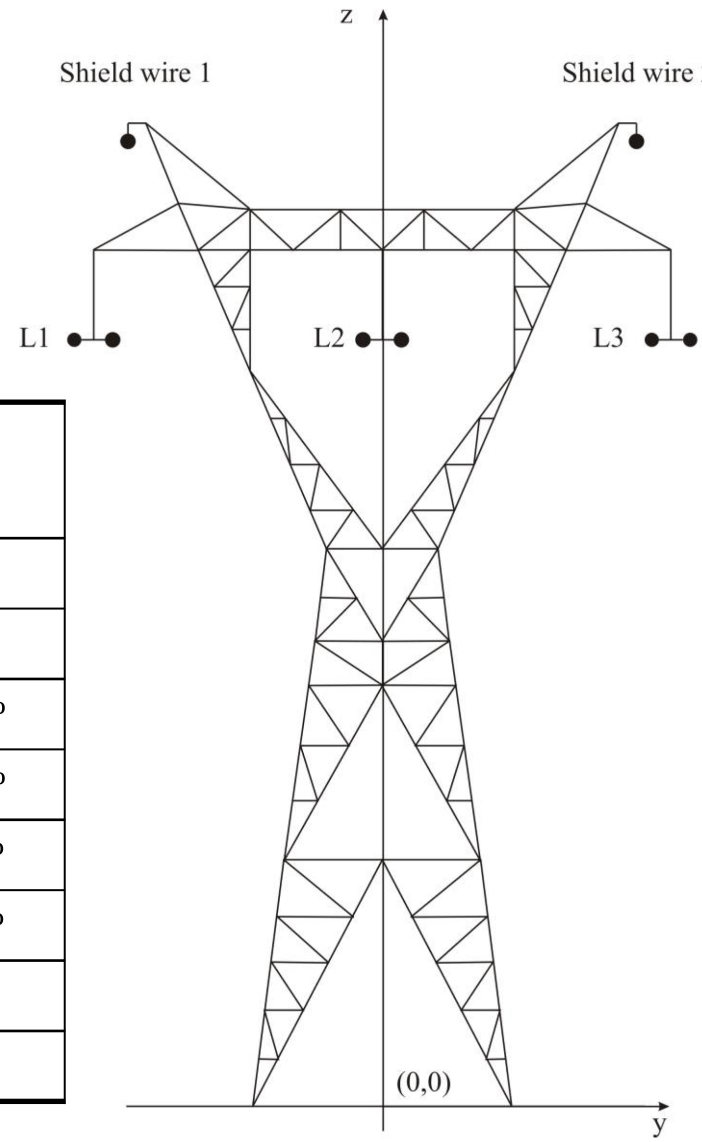
$$d_{ii}=r_{0i} \qquad d_{ij}=\sqrt{\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2} \quad ; \quad i\neq j$$

$$D_{ii}=2\cdot z_i \qquad D_{ij}=\sqrt{\left(y_i-y_j\right)^2+\left(z_i+z_j\right)^2} \quad ; \quad i\neq j$$

Input data of the overhead power line

- 6 phase conductors (AlFe 490/65) and 2 shield wires (Alumweld 19/9)
- 376 m long power line section

	i	y (m)	z_s (m)	$\overline{\Phi}_i$ (kV)	$\bar{I}_i$ (A)
L1	1	-10.7	**12.58**	234.4∠0°	228∠-4.2°
	2	-10.4	**12.58**	234.4∠0°	228∠-4.2°
L2	3	-0.15	**12.58**	234.4∠240°	228∠235.8°
	4	0.15	**12.58**	234.4∠240°	228∠235.8°
L3	5	10.4	**12.58**	234.4∠120°	228∠115.8°
	6	10.7	**12.58**	234.4∠120°	228∠115.8°
SW1	7	-7.44	**19.09**	0∠0°	0∠0°
SW2	8	7.44	**19.09**	0∠0°	0∠0°



Input data of the overhead power line

- The sag of the conductors was taken into account by:

$$z_s = z - \frac{2}{3} \cdot s$$

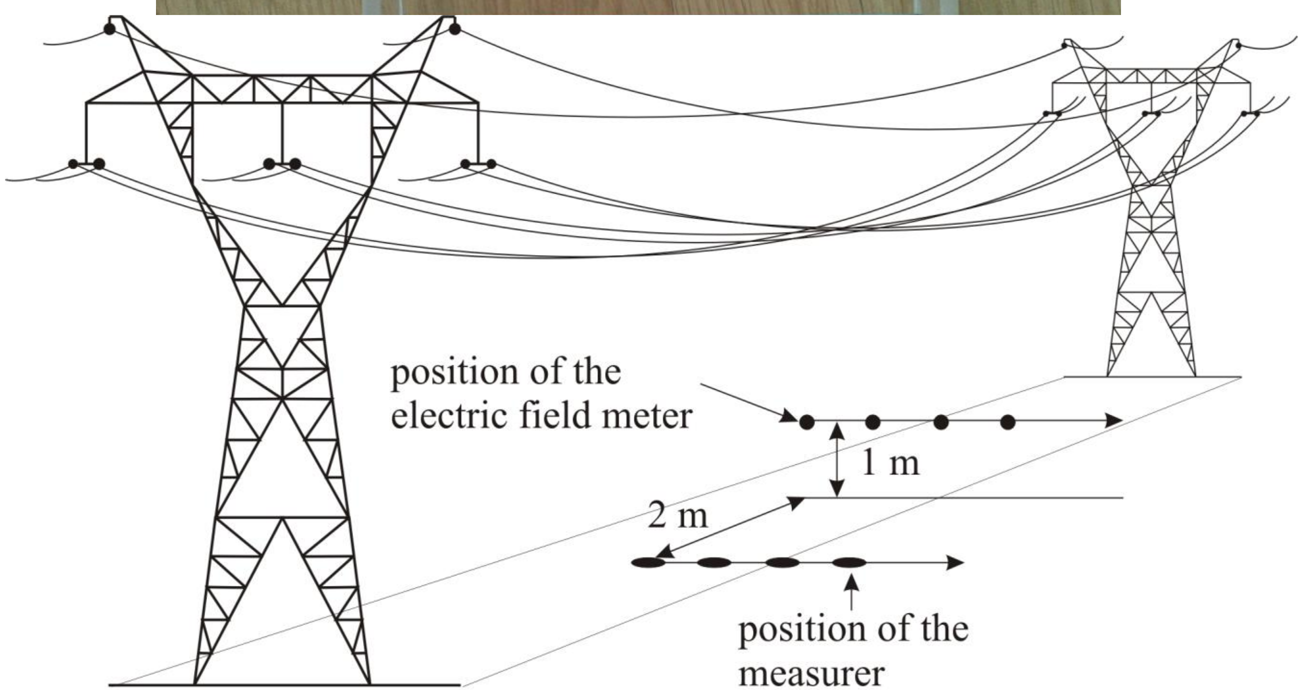
- measurements taken on a clear winter day (10 °C)
- terrain mostly clear except sporadic bushes
- measurements along *y* directed observation profile at $z = 1$ m

- Other input data:

$$\varepsilon_r = 10$$
$$\sigma = 0.1\, S/m$$
$$f = 50\, Hz$$

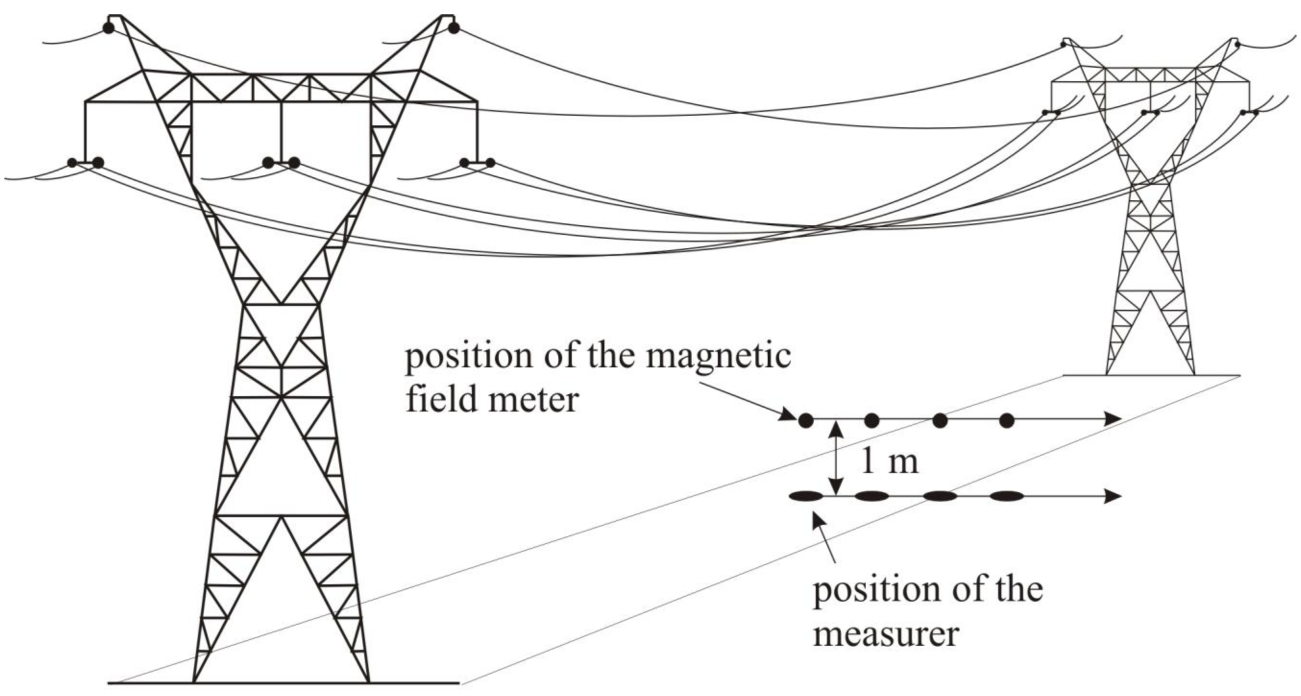
Measuring equipment and procedure

- electric field meter Monroe Electronics Model 238A-1 AC Fieldmeter
- measurer has influence on electric field distribution → measuring probe **must** be used

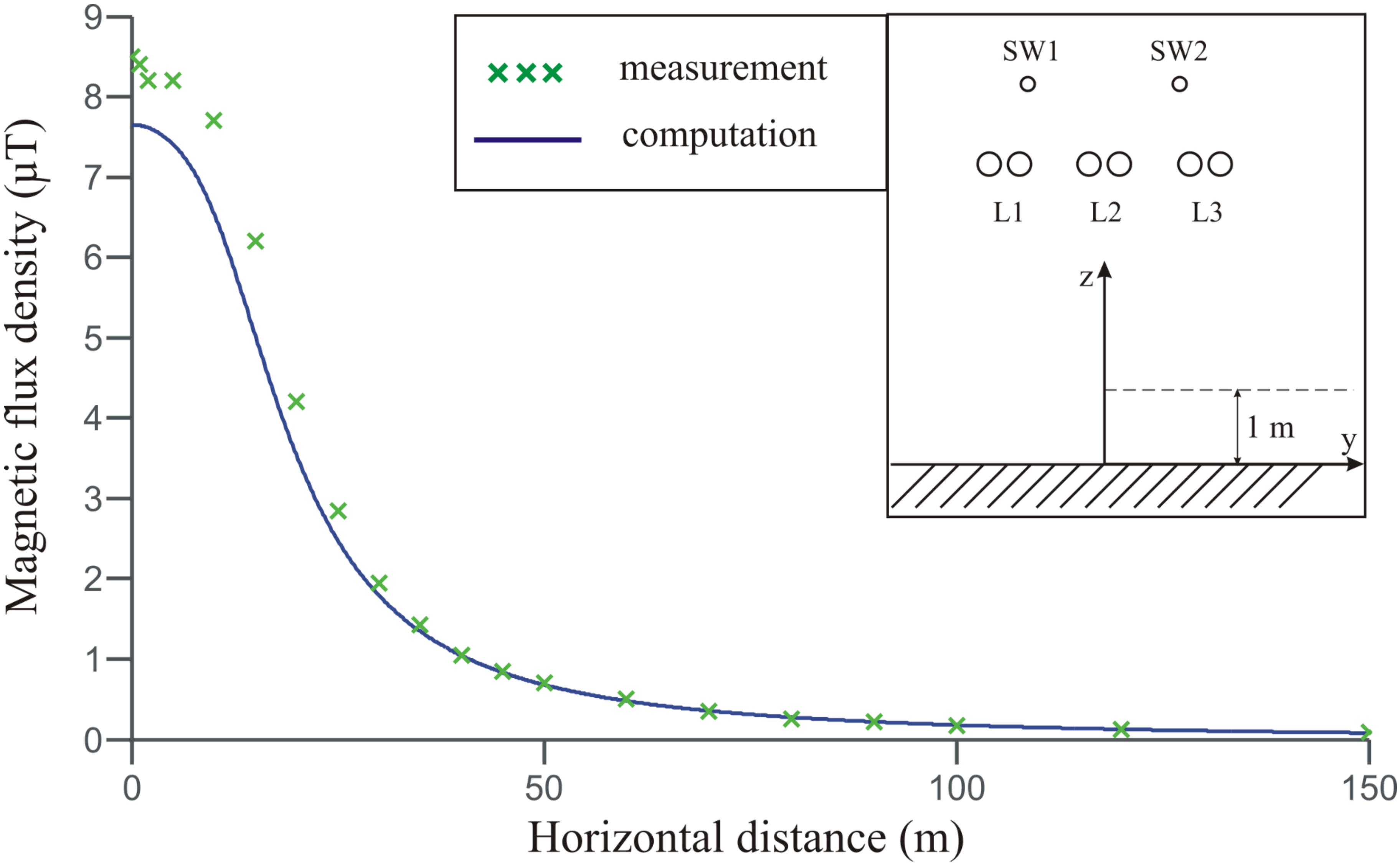


Measuring equipment and procedure

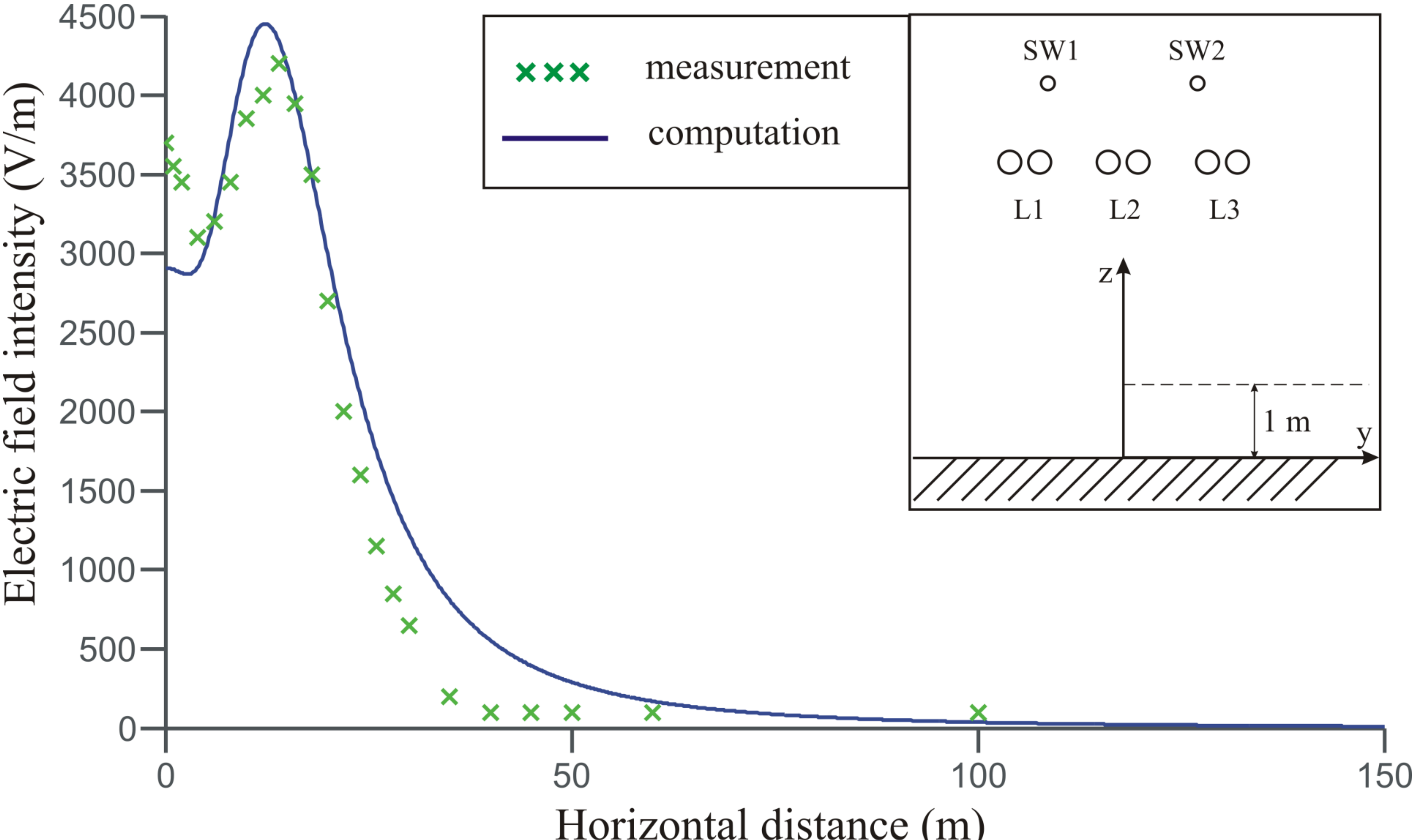
- magnetic field meter Sypris Triaxial ELF Magnetic Field Meter Model 4090
- measurer does not have influence on the magnetic field distribution



Comparison of results – magnetic field



Comparison of results – electric field



Summary

- 2D algorithms for computation of electric and magnetic fields of overhead power lines
- good agreement was found between the computed results and measurements
- better results can be achieved by taking the sag into account (3D algorithm)
- more advanced models are needed for more complicated structures such as electric power substations

Thank you!